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# DESIGN AND ANALYSIS OF FULL-STATE FEEDBACK CONTROLLER FOR A TRACTOR ACTIVE SUSPENSION

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## ABSTRACT

Vehicle suspension systems are needed in modern tractors to improve ride comfort by insulating driver's cabin from road disturbances. Active suspension (AS) systems have the potential to improve both ride quality and handling vibration performance upon use of feedback to control its hydraulic actuator. This gives a capability to the vehicle to continuously adjust itself and response to the varying road conditions. The main objective of this study was to use a full-state feedback approach to design and analysis of AS control system for Kubota M110X tractor to eliminate the transmitted vibrations to the driver's cabin caused by field roughness. The inputs of the system were determined as the control force generated from the hydraulic actuator of the AS and the road disturbances caused by holes and uneven surface. A simulation model was developed to analyze the behavior of the system to disturbances with 0.25m amplitude. Results are included to show the dynamic performance and robustness of the proposed controller in dissipating the corresponding disturbance vibrations for a comfort ride with an instant overshoot of about %12 of the inputs disturbance and a settling time (ST) of 4.36 seconds.

***Keywords**: Active suspension, tractor, controller, state feedback, simulation*

## INTRODUCTION

The desire for more comfortable ride in farm tractors has been a motivation for modern tractor manufacturing industries to design complex electro-hydraulic controls for their active and semi *AS* systems. Mechanical vibration transmitted to tractor's driver caused by the unevenness of the road or soil profile, or moving elements within the machine or implements [1] can cause physiological and psychological harm effects. The classification for roughness has been provided by the International Standardization Organization (ISO) using the Power Spectral Density (PSD) values [2]. According to [3], the endurance limit for human body in vertical acceleration is in the range of 4–8 Hz and root-mean-square (RMS) acceleration less than1 m s-2 [4]. Farm tractors' drivers performing ploughing and harrowing operations are subjected to such vibrations that can cause severe discomfort and injuries. Low back and pain disorders have been reported among tractor drivers due to continuous exposure to whole body vibration [5]. As described by [8], "the main aim of suspension system is to isolate a vehicle body from road irregularities in order to maximize passenger ride comfort and retain continuous road wheel contact in order to provide road holding". While maximizing the tire-to-road contact, a suspension system should minimize the vertical forces transmitted to the driver caused by ground vibration which yields to smaller vertical body accelerations. To this aim, an actuator that is incorporated to an *AS* system applies the control forces to the vehicle body of the tractor for reducing its vertical acceleration to zero. This gives a capability to the vehicle to continuously adjust itself and response to the varying road conditions.

The topic of *AS* control system for road vehicles has been quite challenging in the past years. The very first experimental tractor cab suspension systems were developed by a number of researchers during the 1970s [6]. A comprehensive review on control design of AS systems over the last 20 years has been provided by [7]. Mechanical suspensions including helical spring, with or without shock absorbers and hydro-pneumatic and air suspensions with Nitrogen based gas plus oil filled chamber are commonly used in farm tractors [9]. Different control strategies have been proposed for each of these systems, including linear quadratic regulation (LQR) in combination with nonlinear back-stepping control techniques [10]. These approaches require information of the vertical positions and speeds of the tire and car body. A controller of variable gain that considers the nonlinear dynamics of the suspension system has been proposed by [11]. A simpler controller design to decrease the implementation costs without sacrificing the security and the comfort by using accelerometers for measurements of the vertical movement of the tire and car body has been discussed in [12].

The objective of this work was to design and analyze a full-state feedback AS controller for Kubota M110X tractor in such a way that when rear wheels are subjected to holdings and bumps, like field pot holes, cracks and uneven surfaces, the *AS* system can provide comfort riding by dissipating the resulting oscillations within a ST of less than 5 seconds and overshoot of about 10% of the inputs disturbance.

## MATERIALS AND METHODS

The *AS* model with an electro-hydraulic actuator is shown schematically in Fig. 1 with single wheel and axle connected to the quarter portion of the Kubota M110X tractor body (Fig. 2) through an active spring-damper combination, where $M_1$ and $M_2$ are the tractor mass and the suspension mass, $x_s$ and $x_w$ are the displacement of tractor body and the suspension mass, $k_1$ and $k_2$ are the spring coeeficients, and $b_1$ and $b_2$ are the damper coefficients.

**Figure 1. Kubota M110X - tractor with cabin**

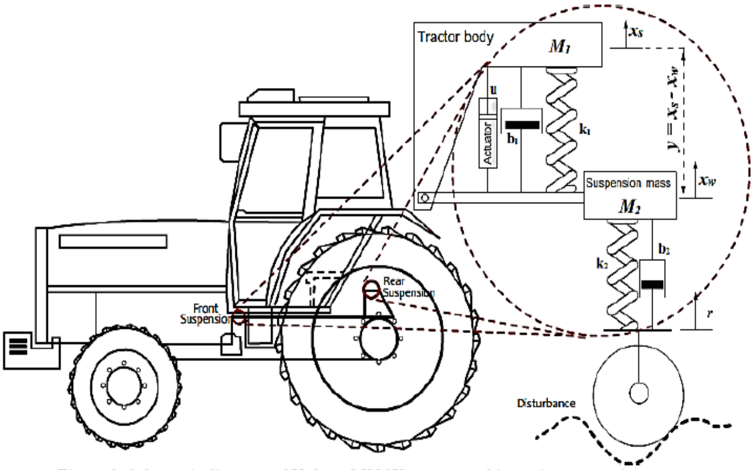


**Figure 2. Schematic diagram of Kubota M110X tractor and its active suspension system**

### Dynamic model

The inputs of the systems are the control force from the actuator, and the field disturbance that is later modeled by a sinusoidal and step function to represents uneven field surface. The equation of vertical motion for the system of mass-spring and shock absorber was written based on Newton's law and are given by two differential equations in (1) and (2).

$$\frac{d^2x_s}{dt^2} = \frac{1}{M_b}\left[b_1\left(\frac{dx_w}{dt} - \frac{dx_s}{dt}\right) + k_a(x_w - x_s) + u\right] \tag{1}$$

$$\frac{d^2x_w}{dt^2} = \frac{1}{M_{us}}\left[b_1\left(\frac{dx_s}{dt} - \frac{dx_w}{dt}\right) + k_a(x_s - x_w) + b_2\left(\frac{dr}{dt} - \frac{dx_w}{dt}\right) + k_2(r - x_w) - u\right] \tag{2}$$

The output of this system is defined as the difference in the displacement change between $x_s$ and $x_w$, hence the control objective is to create control force, $u$, from the actuator in such a way that the output, $y = x_s - x_w$ will be able to track the disturbance, $r$ with an overshoot about 10% and a ST less 5 seconds. The relationship between the two inputs and one output can be developed in Laplace domain by considering the control and disturbance inputs individually. This yields two strictly proper transfer functions that are convertible into two reachable canonical state space representations through the nonhomogeneous differential equation given in (3) and (4) and then setting all the initial conditions to zero. The system states were defined by letting $x_1 = x_s$, $x_2 = \dot{x}_s$, $x_3 = y$ and $x_4 = \dot{y}$. The resulting model is provided in (5) with one input, one output and four states, where $x(t) \in \mathbb{R}^4$ is the state vector, $y(t) \in \mathbb{R}$ is the output vector, $u(t), r(t) \in \mathbb{R}$ are the control and the disturbance input and $[A]_{4\times4}$, $[B]_{4\times1}$, $[C]_{1\times4}$ and $[D]_{1\times1}$ are the system matrix, input matrix, output matrix and the feed-forward matrix respectively. These models are controllable canonical form since the control can enters a chain of integrators to move every state. In addition, based on the Routh-Hurwitz criterion algorithm, both systems were checked and are controllable and observable.

$$y^{(n)} = f(t, u(t), y, \dot{y}, \ddot{y}, \dots, y^{(n-1)}), I.C: y(0) = y_0, \dot{y}(0) = y_1(0), \dots, y^{(n-1)}(0) = y_{n-1}(0)$$

$$\begin{aligned} y^{(4)} = \frac{1}{M_1M_2}[(M_1+M_2)\,u'' + b_2u' + k_2u] \\ - \frac{1}{M_1M_2}\big[(M_1(b_1+b_2)+M_2b_1)y^{(3)} + (M_1(k_1+k_2)+M_2k_1+b_1b_2)y'' + (b_1k_2+b_2k_1)y' \\ + (k_1k_2)y\big] \end{aligned} \tag{3}$$

$$\begin{aligned} y^{(4)} = \frac{1}{M_1M_2}[-(M_1b_2)\,r'' - M_1k_2r'] \\ - \frac{1}{M_1M_2}\big[(M_1(b_1+b_2)+M_2b_1)y^{(3)} + (M_1(k_1+k_2)+M_2k_1+b_1b_2)y'' + (b_1k_2+b_2k_1)y' \\ + (k_1k_2)y\big] \end{aligned} \tag{4}$$

$$\begin{cases}\dot{x}_1(t) = Ax(t) + B.u(t) \\ \dot{x}_2(t) = Ax(t) + B.r(t)\end{cases}, \quad \begin{cases} y_1(t) = C_1.x(t) \\ y_2(t) = C_2.x(t)\end{cases}$$

$$A = \begin{bmatrix} -\frac{M_1(b_1+b_2)+b_1M_2}{M_1M_2} & -\frac{M_1(k_1+k_2)+M_2k_1+b_1b_2}{M_1M_2} & -\frac{b_1k_2+b_2k_1}{M_1M_2} & -\frac{k_1k_2}{M_1M_2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \tag{5}$$

$$C_1 = \frac{1}{M_1M_2}[0 \quad M_1+M_2 \quad b_2 \quad k_2], \qquad C_2 = \frac{1}{M_1M_2}[-M_1b_2 \quad M_1k_2 \quad 0 \quad 0]$$

**Controller design**

The open loop poles of the transfer functions corresponding to the dynamics described in (1) and (2) are shown by means of root locus plots in figures 3 and 4 to check the behavior of the model. Both systems are observed to be stable, however the open-loop response, especially for the second system is highly oscillatory due to the existence of imaginary components in the poles. To dissipate these oscillations, a controller was designed according to the block diagram in Fig. 5, assuming that all the states are measurable. Utilization of vertical displacement, speed and acceleration sensors in experimental and commercial vehicle platforms has been discussed by [13]. Laser sensors can be used to measure the state variables $x_1 = x_s$ and $x_3 = y$ for implementation of the controller. For measuring the other two variables, $x_2 = \dot{x}_s$ and $x_4 = \dot{y}$, accelerometers or other types of sensors are not needed since these variables can be estimated with the use of integral reconstruction from the control input and output. A new state, $x_5 = \int y(t)dt$, was added to the system in order to achieve zero dynamic. This integral action produces zero error if the closed loop system reaches steady state. The closed-loop state-space model for the full state feedback controller is provided in (6) which show that after the tractor tire is subjected to a field disturbance, it will ultimately reach to equilibrium point. The values of the control matrix, $[K]_{1\times5} = [250 \quad 500 \quad 300 \quad 200 \quad 150]$ were adjusted based on simulation trial and error approach. The controller was designed to feedback the five states, $[x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5] = \left[x_s. \frac{dx_s}{dt}.(x_s - x_w) = x_s - x_w \frac{d}{dt}(x_s - x_w) \int (x_s - x_w)dt\right]^T$.

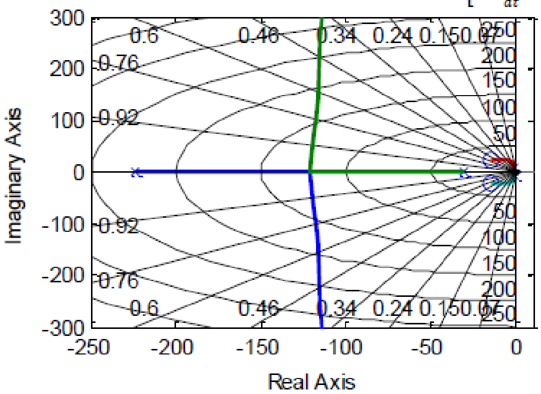


Figure 3. Root locus system 1

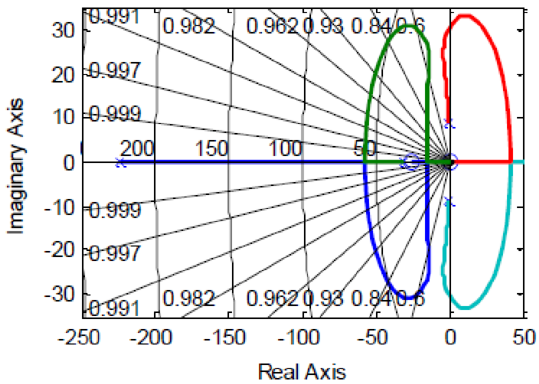


Figure 4. Root locus system 2

$P_1 = -224.24$, $P_2 = -31.14$, $P_3 = -0.44 + 8.95i$, $P_4 = -0.44 - 8.95i$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \left( \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{b_1 b_2}{M_1 M_2} & 0 & \frac{b_1^2}{M_1^2} + \frac{b_1^2}{M_1 M_2} + \frac{b_1 b_2}{M_1 M_2} - \frac{k_1}{M_1} & -\frac{b_1}{M_1} & 0 \\ \frac{b_2}{M_2} & 0 & -\frac{b_1}{M_1} - \frac{b_1}{M_2} - \frac{b_2}{M_2} & 1 & 0 \\ \frac{k_2}{M_2} & 0 & -\frac{k_1}{M_1} - \frac{k_1}{M_2} - \frac{k_2}{M_2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ \frac{1}{M_1} & \frac{b_1 b_2}{M_1 M_2} \\ 0 & -\frac{b_2}{M_2} \\ \frac{M_1+M_2}{M_1 M_2} & -\frac{k2}{M2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix}1\\0\end{bmatrix} \begin{bmatrix}K_1 & K_2 & K_3 & K_4 & K_5\end{bmatrix} \right) \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M_1} & \frac{b_1 b_2}{M_1 M_2} \\ 0 & -\frac{b_2}{M_2} \\ \frac{M_1+M_2}{M_1 M_2} & -\frac{k2}{M2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix}U\\R\end{bmatrix} \qquad (6)$$

$$y = \begin{bmatrix}0 & 0 & 1 & 0 & 0\end{bmatrix} \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$$

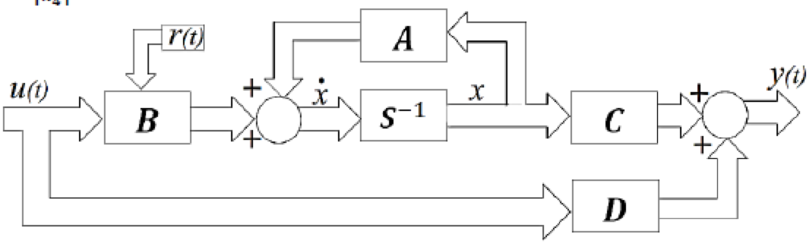


**Figure 5. State space control representation block diagram**

## RESULTS AND DISCUSSIONS

The simulation model was designed in by means of MATLAB-Simulink© with block diagram provided in Fig. 6. The road roughness and disturbances were simulated by step (representing uneven surface) and sinusoidal functions (representing pot holes and bumps) with 0.25 m amplitude and 0.5 seconds of duration (Fig. 7) and were then programmed into the Simulink blocks by means of signal generator. The numerical values of the suspension model parameters for Kubota M110X tractor are proposed as follow; sprung mass $M_1$ = 700 kg, un-sprung mass $M_2$ = 90 kg, spring stiffness $k_1$ = 62000 N/m, $k_2$ = 570000 N/m, damper constant $b_1$ = 500 N.s/m and $b_2$ = 22500 N.s/m. The behavior of the open-loop (passive) systems to step and sinusoidal disturbance with 0.25m amplitude are shown in figures 8 to 11. It can be seen that the ST for the seat under the passive system is 8.47 seconds with infinite overshoot. The closed-loop (active) system response to step disturbance is shown in Fig. 11 with ST of 4.36 seconds. To verify that the designed full-state feedback controller archives the desired objectives for the two disturbance types (step input for uneven surface and sinusoidal input for pot holes and bumps), the open-loop (passive) and closed-loop (active) simulation responses were compared and their plots are shown in figures 12 through 15. It can clearly be observed that for both disturbance types, the active system has a much faster stabilization than the passive system. Plots of control effort, $u(t)$, $x_1(t) = x_s$, $x_2(t) = \dot{x}_s$ and $x_4(t) = \dot{y}$ are also shown in figures 16 through 19 respectively for a time interval of 10 seconds.

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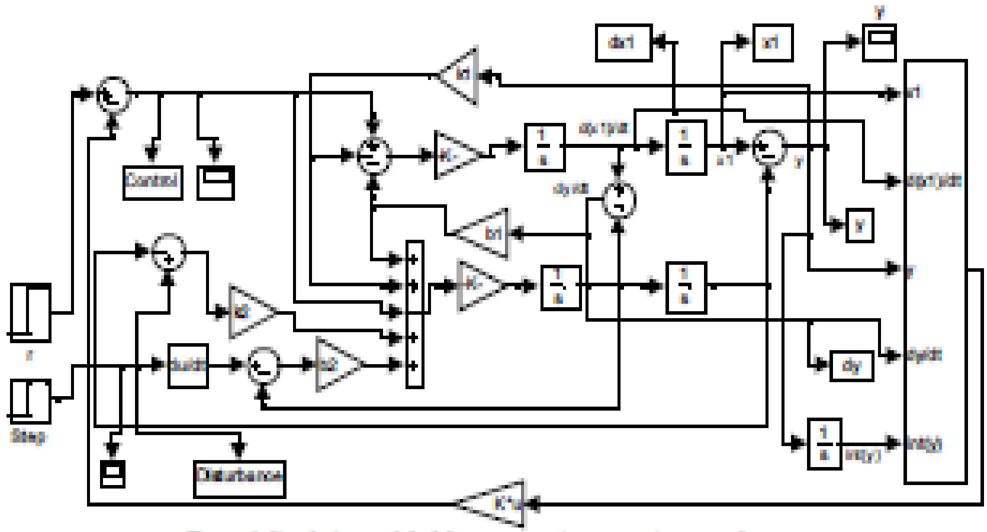


Figure 6. Simulation model of the tractor active suspension control system

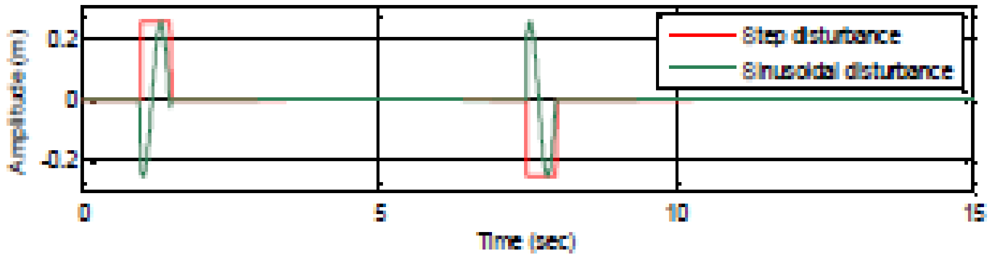


Figure 7. Simulation of road roughness disturbances

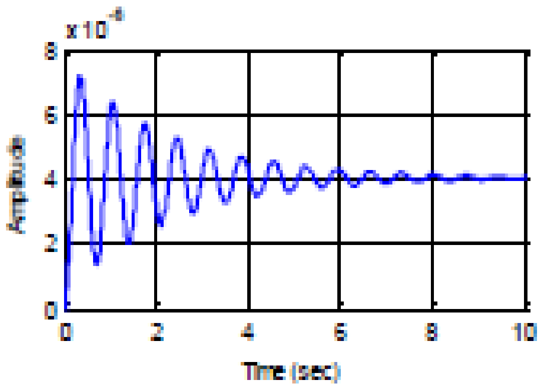


Figure 8. Step response of system 1, step=0.25m, ST=8.47s, overshoot= 78.4%

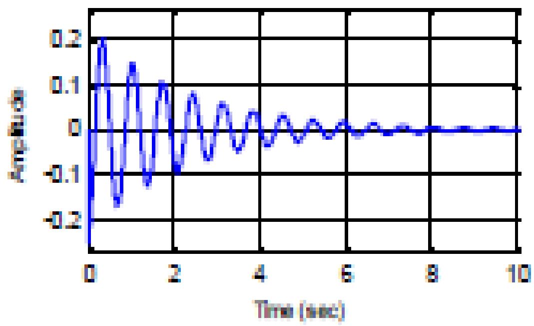


Figure 9. Step response of system 2, amplitude=0.25m, settling time=8.47s, infinite overshoot

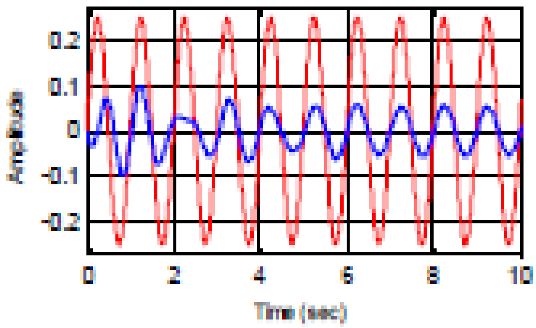


Figure 10. Sinusoidal response of system 1, amplitude=0.25m

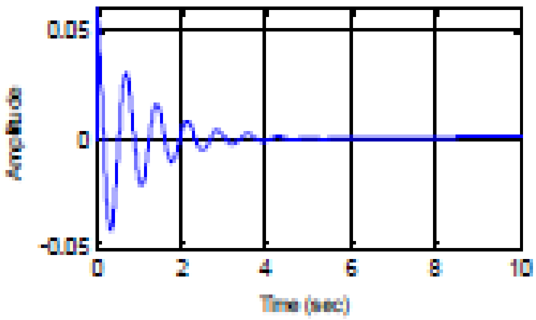


Figure 11. Closed-loop step response, settling time=4.36 s

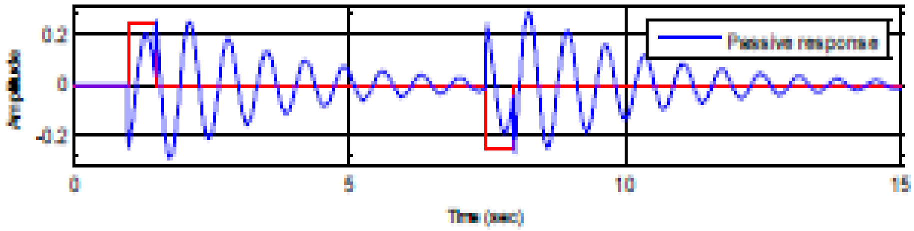


**Figure 12. Simulation of the uncontrolled response to two step disturbance, with duration=0.5 seconds and amplitude of +/-0.25 m**

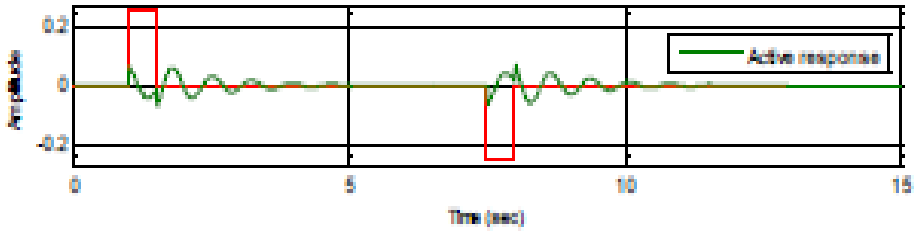


**Figure 13. Simulation of the controlled response to two step disturbance, with duration=0.5 seconds and amplitude of +/-0.25 m**

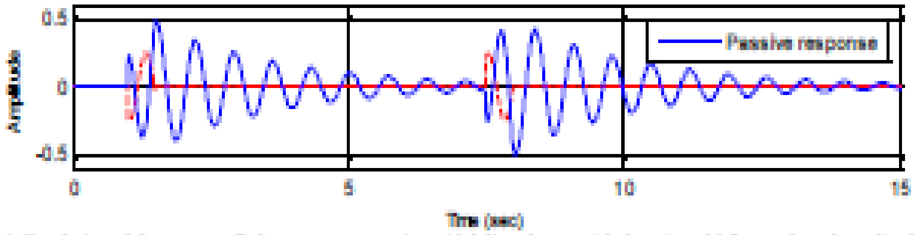


**Figure 14. Simulation of the uncontrolled response to two sinusoidal disturbance with duration of 0.5 seconds and amplitude of 0.25 m**

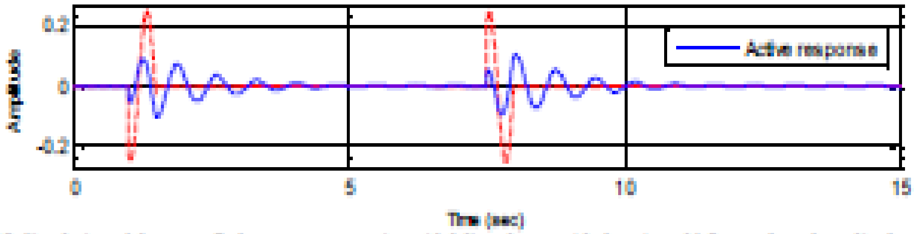


**Figure 15. Simulation of the controlled response to two sinusoidal disturbance with duration of 0.5 seconds and amplitude of 0.25 m**

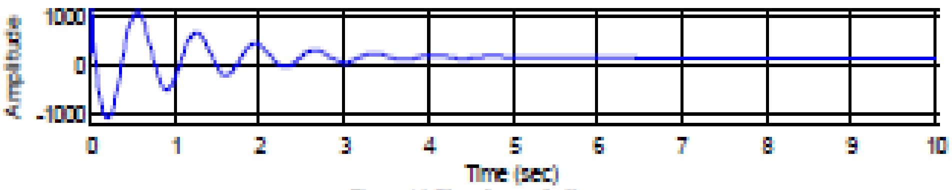


**Figure 16. Plot of control effort**

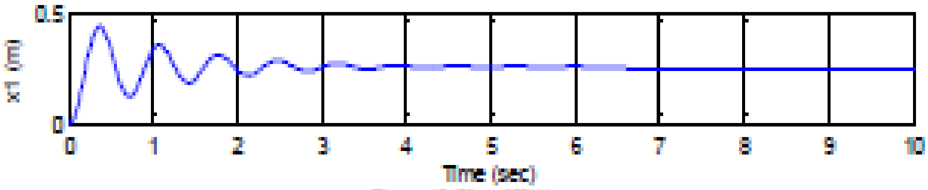


**Figure 17. Plot of X1(t)**

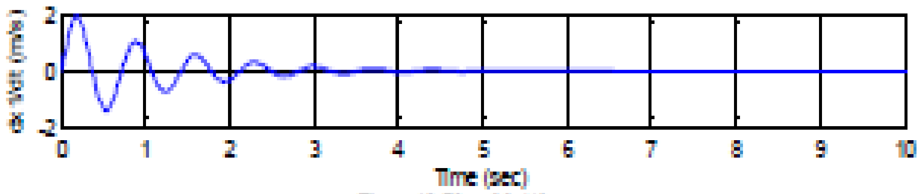


Figure 18. Plot of dx1/dt

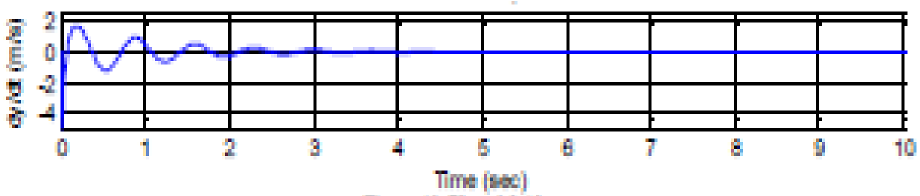


Figure 19. Plot of dy/dt

## CONCLUSIONS

This paper discussed the general conditions for suspension control and various control concepts in AS systems. It also presented a full-state feedback approach of robust active vibration control schemes for electro-hydraulic AS systems of Kubota M110X tractor. The control objective was to attenuate the vibrations induced by exogenous disturbances excitations due to irregular road surfaces. These disturbances were modeled by step and sinusoidal input functions with amplitude of 0.25 m to simulate uneven surfaces and bumps. One advantage of this design is its measurement requirement to position sensors only. To implement derivatives of the states, integral reconstruction was employed for structural estimates of the time derivatives. The simulation results show considerable differences between the results of passive and the design schemes of AS system. The final simulation results showed dissipation of the vertical oscillations within a significant shorter time than the uncontrolled system. It can also be seen that the designed controller has a fast stabilization with amplitude in acceleration and speed of the body of the tractor. Finally, the robustness of the controllers to stabilize to the system before the unknown disturbance is verified.

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