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Development and testing of generalized wavelet-neural network based evapotranspiration models with limited climatic data in India

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Abstract

In this paper, generalized wavelet-neural network based (GWNN) models were developed for estimating reference evapotranspiration (ET_o) corresponding to Hargreaves (HG) method for different agro-ecological regions (AERs) in India. The daily pooled climate data (minimum and maximum air temperatures and extra terrestrial radiation) from 15 different locations under four different AERs in India are used as an input to GWNN models and the target consists of the FAO-56 PM estimated ET_o. Further, the GWNN models were applied to 15 individual model development and 10 different model testing locations to test the generalizing capability. Comparison of developed GWNN models was made with the classic generalized artificial neural network (GANN), generalized linear regression (GLR), generalized wavelet regression (GWR), and HG method to test the superiority of one model over the other. The performance indices used for comparison include root mean squared error, Nash-Sutcliffe efficiency, ratio of average output to average target ET_o values, and relative percentage. Based on the comparisons, it is concluded that the GWNNs followed by GANN models performed better than GWR and GLR models for four AERs. The testing results suggest that the GWNN and GANN models have better generalizing capabilities than the GWR and GLR for all region locations.

Keywords: neural networks, discrete wavelet transformation, evapotranspiration, agro-ecological regions

INTRODUCTION

Reference evapotranspiration (ET_o) is one of the significant components of the hydrologic cycle. Accurate estimation of ET_o is important for carrying out many water resources and hydrological studies. There exist a number of direct and indirect ET_o estimation methods. But, most of the existing ET_o estimation methods have the number of limitations (Adamala et al., 2014). To avoid the limitations of existing ET_o models, the artificial neural networks (ANNs) are used in ET_o modeling. These ANNs can model the complex non-linear ET_o without having a complete understanding of it. The climate data that needed for accurate estimation of ET_o might cope with non-stationarity. A wavelet transformation (WT) serves as an effective tool for accurately modeling ET_o using various non-stationary hydro-climatic variables by locating the irregular spatial and temporal distributed multi-scale features of data. Over the years, a number of studies were reported in the literature on the application of wavelet neural networks (WNNs; combination of ANNs and WT) for modeling different hydro-climatic variables, especially ET_o (Partal, 2009; Kisi, 2011; Cobaner, 2013; Evrendilek, 2014; Falamarzi et al., 2014).

The WNN model developed for one location might only be useful in the developed location unless the external generalizability is evaluated, which is not done in most of the studies. If these WNN models are only accurate for the training locations, their real

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applicability is limited to local emergency cases, like breakdowns in the data acquisition system. Hence, there is a need to develop generalized WNN (GWNN) models which are not only applicable for model training locations but also for outside the training locations. This can be achieved by considering pooled data of various locations, which have properties of both spatial and altitudinal variations during training (Adamala et al., 2015). Further, in a developing country like India with higher spatial variation in climate, the required climatic data for ET_0 estimation may be extremely hard to obtain at all locations due to the difficulty in observation. The most readily available data for India may be the maximum air temperature (T_{max}) and minimum air temperature (T_{min}). This shows the need of developing GWNN models for with limited input data i.e. corresponding to Hargreaves (HG) method. The objectives of this study are formulated as: (i) to develop the GWNN models corresponding to HG method for four agro-ecological regions (AERs: semi-arid, arid, sub-humid, and humid) of India, (ii) to test the generalizing capability of GWNN models with the model development and model testing locations, and (iii) to compare the developed GWNN models with the generalized ANN (GANN), generalized linear regression (GLR), generalized wavelet regression (GWR), and conventional HG method.

STUDY AREA AND CLIMATE DATA

The study area consists a total of 25 different meteorological locations in India. The data sample consists of daily climate data of T_{min} , T_{max} , and extra terrestrial radiation (R_a). Due to the unavailability of measured lysimeter ET_0 data for the selected study locations, it was estimated by the FAO-56 PM method. Table 1 presents information related to latitude, longitude, altitude, and observation periods for the chosen locations.

Table 1. Characteristics and summary of study locations.

	Location	Lat. (°N)	Lon. (°E)	Alt. (m)	Role ^a	Period
Semi-arid	Parbhani	19°08'	76°50'	423	Tr, V, Ts	2001-05
	Solapur	17°41'	75°56'	25	Tr, V, Ts	2001-05
	Bangalore	12°58'	77°35'	930	Tr, V, Ts	2001-05
	Kovilpatti	9°10'	77°52'	90	Tr, V, Ts	2001-05
	Udaipur	25°21'	74°38'	433	Tr, V, Ts	2001-05
	Kanpur	26°26'	80°22'	126	Ts	2004-05
	Anand	22°33'	72°58'	45	Ts	2002-05
	Akola	20°42'	77°02'	482	Ts	2001-03
Arid	Anantapur	14°41'	77°37'	350	Tr, V, Ts	2001-05
	Hissar	29°10'	75°44'	215	Tr, V, Ts	2001-05
	Bijapur	16°49'	75°43'	594	Ts	2001-04
Sub-humid	Raipur	21°14'	81°39'	298	Tr, V, Ts	2001-05
	Faizabad	26°47'	82°08'	133	Tr, V, Ts	2001-05
	Ludhiana	30°56'	75°52'	247	Tr, V, Ts	2001-05
	Ranichauri	30°52'	78°02'	1600	Tr, V, Ts	2001-05
	Jabalpur	23°09'	79°58'	393	Ts	2002-05
	Samastipur	25°53'	85°48'	52	Ts	2004-05
	Bhubaneshwar	20°15'	85°50'	25	Ts	2002-05
	Ranchi	23°17'	85°19'	625	Ts	2005
Rakh Dhiansar	32°39'	74°58'	332	Ts	2005	
Humid	Palampur	32°06'	76°03'	1291	Tr, V, Ts	2001-05
	Jorhat	26°47'	94°12'	86	Tr, V, Ts	2001-05
	Mohanpur	21°52'	87°26'	10	Tr, V, Ts	2001-05
	Dapoli	17°46'	73°12'	250	Tr, V, Ts	2001-05
	Thrissur	10°31'	76°13'	26	Ts	2001-04

^aTr: Train; V: Validation; and Ts: Test

DESCRIPTION OF MODELS AND EVALUATION

Hargreaves ET_o estimation method

The temperature based Hargreaves method that demands fewer climate data for the estimation of ET_o is described as below:

$$ET_o = 0.0023R_a \sqrt{TD} (T_{avg} + 17.8) \quad (1)$$

where ET_o = reference evapotranspiration (mm day⁻¹); T_{avg} = average daily air temperature at 2 m height (°C); T_{max} = maximum daily air temperature at 2 m height (°C); T_{min} = minimum daily air temperature at 2 m height (°C); TD = difference between T_{max} and T_{min} (°C); R_a = extraterrestrial solar radiation (function of latitude and day of the year) (MJ m⁻² day⁻¹).

Artificial neural network (ANN)

ANN is a massively parallel distributed processor that has a natural propensity for storing experiential knowledge and making it available for use. The ANN employed in this study consists of an input (*i*), hidden (*j*), and an output (*k*) layers with the interconnection weights *w_{ij}* and *w_{jk}* between layers of neurons. These can be represented as:

$$y = \phi \left[\sum_{i=0}^n w_i x_i \right] \quad (2)$$

where *x* = input vector; *y* = output vector; *w* = set of adaptive parameters (weights); *n* = number of elements in the input vector; and ϕ = sigmoidal activation function.

Wavelet transform (WT)

During the past few decades, the WT appears to be a more effective tool than the Fourier transform (FT) that do not provide an accurate time-frequency analysis for non-stationary signals. The WT analyzes the distorted time-series into different time-frequency scales by detecting the disturbances present in the data. The WT uses two basic functions, i.e. the wavelet (φ) and scaling (Φ) to perform simultaneously the multi-resolution analysis decomposition and reconstruction of the measured time-series data. The high-frequency components (details) are generated by φ whereas Φ generates the low-frequency components (approximations) of the distorted time-series. Discrete wavelet transform (DWT) which is a feature extraction technique translates each time-series from the time domain to the time/frequency domain without losing any information about the instant when the change occurred.

$$DWT_{\Psi} x(m, n) = \sum_k x_k \Psi_{m,n}^*(k) \quad (3)$$

$$\Psi_{m,n}^*(k) = \frac{1}{\sqrt{a_0^m}} \Psi^* \left(\frac{k - nb_0 a_0^m}{a_0^m} \right) \quad (4)$$

where Ψ = mother wavelet function; * = a complex conjugate; a_0^m = scaling factor; $nb_0 a_0^m$ = shifting factor; *m* and *n* = scaling and sampling numbers, respectively.

Development of GWNN models

For developing different GWNN models for daily ET_o estimation, the code was written using Matlab programming language. The input time series (T_{min}, T_{max}, and R_a) was decomposed into different sub-time series components using 'daubechies (db10)' wavelet function. The pooled climate data for the four AERs is decomposed into ten detailed (D1-D10) and one approximated (A10) components using DWT algorithm. The correlation coefficients of the reconstructed climate data with the FAO-56 PM ET_o are shown in Table 2. These decomposition level modes are represented as 2-day (D1), 4-day (D2), 8-day (D3), 16-day (D4), 32-day (D5), 64-day (D6), 128-day (D7), 256-day (D8), 512-day (D9), and 1024-day (D10). Among D1-D10 modes, D8 mode has the highest correlation and D1 and D2 have lowest correlations for all locations with the ET_o. Even, the correlation of D8 level was better than the

Approximated mode (A10). Therefore, the annual scale has the greater periodical influence on ET_0 estimation for this study area. The ineffective components (shown as 'bold' in Table 2) are eliminated from the reconstructed series to constitute effective inputs to GWNN models. Further, these models were compared with the GLR, GWR, and GANN models. The input to GLR and GANN models includes the original climatic time-series data, whereas DWT decomposed effective sub-times series data was used as an input to GWR models. The GWR model is a conjunction of DWT and GLR models.

Table 2. The correlation coefficients of reconstructed climate data with ET_0 for four AERs.

AER	Variable	Approximation					Details					
		A10	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
Semi-arid	T_{max}	0.40	0.08	0.11	0.12	0.12	0.10	0.13	0.29	0.55	0.14	0.15
	T_{min}	0.40	0.03	0.03	0.02	0.03	0.01	0.06	0.17	0.42	0.07	0.11
Arid	T_{max}	0.34	0.04	0.11	0.12	0.11	0.11	0.13	0.19	0.66	0.03	-0.03
	T_{min}	0.36	0.02	0.06	0.07	0.05	0.06	0.09	0.27	0.61	0.08	-0.03
Sub-humid	T_{max}	0.03	0.07	0.11	0.11	0.13	0.10	0.09	0.31	0.78	0.12	0.02
	T_{min}	0.00	0.03	0.05	0.05	0.06	0.05	0.06	0.23	0.69	0.11	0.03
Humid	T_{max}	0.16	-0.02	0.09	0.17	0.18	0.14	0.13	0.37	0.58	0.18	0.05
	T_{min}	0.12	0.00	0.03	0.05	0.06	0.02	0.01	0.30	0.51	0.14	0.01

For the development of GWNN models for four AERs, locations having daily data for the period 2001-05 (1826 patterns) were chosen. The locations with 'Tr, V, Ts' role (Table 1) was used to develop GWNN models. For these locations, 70% and 30% of data for the period of 2001-04 (1461 patterns) were used for training and validation, respectively. The data for the year 2005 were used for model testing. The data were pooled from (Parbhani, Solapur, Bangalore, Kovilpatti, and Udaipur), (Anantapur and Hissar), (Raipur, Faizabad, Ludhiana, and Ranichauri), and (Palampur, Jorhat, Mohanpur, and Dapoli) locations to develop GWNN models for semi-arid, arid, sub-humid, and humid regions, respectively. To test the generalizing capability of the GWNN models, the data from remaining locations that were not used during model development which consist different observation periods were used. The locations with only 'Ts' role (Table 1) was used to test the generalizing capability of the developed models (model testing locations).

Performance evaluation

The root mean squared error (RMSE), Nash-Sutcliffe efficiency (NSE), and the ratio of average output to the average target ET_0 values (R_{ratio}) are used to evaluate the performance of various models. The expressions for these indices are described below:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (T_i - O_i)^2} \quad (5)$$

$$NSE = \left[1 - \frac{\sum_{i=1}^n (T_i - O_i)^2}{\sum_{i=1}^n (T_i - \bar{T})^2} \right] * 100 \quad (6)$$

$$R_{ratio} = \frac{\bar{O}}{\bar{T}} \quad (7)$$

$$RP = \left(\frac{RMSE_1 - RMSE_2}{RMSE_1} \right) * 100 \quad (8)$$

where T_i and O_i = target and output values, respectively; n = number of data points; \bar{O} and \bar{T} = average of output and target values, respectively, $RMSE_1$ = RMSE of conventional method; $RMSE_2$ = RMSE of higher-order models.

SIMULATION RESULTS AND DISCUSSION

This section presents the best achieved results of GWNN models corresponding to HG method under four AERs during testing. To find the accuracy of GWNN models, these were compared with the GLR, GWR, and GANN models. Table 3 shows the performance of GLR, GWR, GANN, and GWNN models in terms of RMSE, NSE, and R_{ratio} under four AERs during testing (2005 year pooled data). Among the all developed models, the GWNN models are the best models with the lowest RMSE (mm day⁻¹) values of 0.658, 0.925, 0.659, and 0.628 for semi-arid, arid, sub-humid, and humid regions, respectively. Here, it is worth to mention that the R_{ratio} is used only to know whether the models overestimated ($R_{ratio}>1$) or underestimated ($R_{ratio}<1$) simulated ET_o values. The performance results of conventional HG method with reference to FAO-56 PM are also shown in Table 3 for the 2005 year pooled data. In some areas, data of the climatic parameters other than air temperature are not available; in such cases practicing engineers should use either GWNN or GANN models for accurate estimation of ET_o instead of other methods.

The developed GWNN, GANN, GWR, and GLR models were also compared with the conventional HG method to find the accuracy of former models over conventional methods. The GWNN and GANN models performed 17-38% better than the conventional HG method for all AERs. The GLR models performed less or equal to the conventional HG method for all AERs. Therefore, GWNN models will have a greater potential application in modeling ET_o under different AERs of India.

Table 3. Performance statistics of generalized models under four AERs during testing.

Model	Semi-arid				Arid				Sub-humid				Humid			
	RMSE	NSE	R_{ratio}	RP'	RMSE	NSE	R_{ratio}	RP'	RMSE	NSE	R_{ratio}	RP'	RMSE	NSE	R_{ratio}	RP'
HG	1.062	54.68	1.139	-	1.247	65.98	0.919	-	1.058	67.02	1.172	-	0.959	44.75	1.199	-
GLR	1.052	57.81	1.015	0.94	1.257	65.43	1.015	-0.80	1.075	65.96	1.027	-1.61	0.892	52.28	1.022	6.99
GWR	1.040	58.76	1.014	2.07	1.236	66.58	1.014	0.88	1.063	66.71	1.029	-0.47	0.877	53.83	1.020	8.55
GANN	0.682	82.26	1.010	35.78	1.023	77.09	0.996	17.96	0.671	87.06	0.992	36.58	0.663	73.62	0.993	30.87
GWNN	0.658	83.47	1.006	38.04	0.925	81.26	0.996	25.82	0.659	87.40	0.998	37.71	0.628	76.32	0.993	34.52

Note: RMSE = mm day⁻¹; R_{ratio} = dimensionless; RP = %; and NSE = %; *RP is the relative percentage change in RMSE of GLR, GWR, GANN, and GWNN models over HG method

Testing generalizing capability of WNN models

The generalizing capability of GLR, GWR, GANN, and GWNN models was tested with 'model development locations' and 'model testing locations'. In order to highlight the necessity of developing different ANN and WNN models, it is necessary to first show the results obtained using conventional ET_o methods (HG). Table 4 shows the performance statistics of GANN and GWNN models in terms of RMSE, NSE, and R_{ratio} with the model development locations under four AERs. The performance of GWNN models during testing the generalizing capability with the model development locations was best.

Table 5 illustrates the performance statistics of GANN and GWNN models in terms of RMSE, NSE, and R_{ratio} with the model testing locations under four AERs. The performance of best performed above models was compared with the corresponding conventional ET_o methods. As expected, the generalizing capability of GANN and GWNN models was better as compared to HG method for all locations. The efforts in developing the complex GANN and GWNN models were justified with their superior performance in estimating accurate ET_o . The above results are quite encouraging and suggest the usefulness of GWNN modeling techniques as an alternative to GANN, GLR, GWR, and conventional estimation approaches for accurate estimation of ET_o .

Table 4. Performance of generalized models for model development locations.

AER	Location	HG			GANN			GWNN		
		RMSE	NSE	R _{ratio}	RMSE	NSE	R _{ratio}	RMSE	NSE	R _{ratio}
Semi-arid	Parbhani	1.057	56.44	1.180	0.649	83.59	0.987	0.925	81.26	0.996
	Solapur	1.001	57.34	1.148	0.708	78.66	0.977	0.759	75.47	0.975
	Bangalore	1.018	17.82	1.013	0.664	65.05	0.981	0.691	62.07	0.983
	Kovilpatti	1.785	- 34.09	1.118	1.026	54.00	1.039	1.163	43.07	0.924
Arid	Udaipur	1.025	70.00	1.017	0.785	82.41	1.026	1.257	54.92	0.817
	Anantapur	1.392	40.29	0.983	1.094	63.15	0.964	1.394	40.11	0.927
	Hissar	1.223	61.15	1.065	0.839	81.73	0.988	1.276	57.75	0.813
Sub-humid	Raipur	1.122	59.01	1.036	0.713	82.01	0.976	0.738	82.23	0.964
	Faizabad	1.265	45.86	0.908	0.623	86.87	0.992	0.641	86.08	0.989
	Ludhiana	1.078	71.64	0.938	0.659	89.56	1.008	0.621	90.59	1.032
	Ranchi	1.110	31.99	0.715	0.602	76.48	1.067	0.541	83.80	1.021
Humid	Palampur	0.759	71.54	0.964	0.691	77.36	0.996	0.611	81.51	1.010
	Jorhat	1.370	- 78.84	0.689	0.706	54.24	0.975	0.613	64.13	0.967
	Mohanpur	1.801	- 90.52	0.627	0.647	75.58	0.963	0.667	73.79	0.977
	Dapoli	1.129	- 9.915	1.263	0.621	65.72	0.909	0.738	53.01	0.926

Table 5. Performance statistics of generalized models with the model testing locations.

AER	Location	HG			GANN3			GWNN3		
		RMSE	NSE	R _{ratio}	RMSE	NSE	R _{ratio}	RMSE	NSE	R _{ratio}
Semi-arid	Kanpur	1.058	69.99	1.149	0.855	79.99	1.051	1.212	60.57	0.845
	Anand	1.146	39.24	1.187	0.619	78.26	0.981	0.737	74.84	0.911
	Akola	1.519	62.14	1.004	1.097	80.36	0.968	1.692	53.01	0.815
Arid	Bijapur	1.023	33.77	1.172	0.746	67.68	0.940	0.919	46.48	0.904
	Jabalpur	1.208	51.16	1.209	0.681	85.78	0.990	0.709	83.16	0.984
Sub-humid	Samastipur	0.942	64.45	1.095	0.692	80.25	0.993	0.675	81.67	0.998
	Bhubaneshwar	1.007	62.49	1.082	0.871	72.58	0.982	0.804	76.09	0.982
	Ranichauri	1.277	30.52	1.301	0.571	75.45	0.981	0.632	75.96	0.985
Humid	Rakh Dhiansar	1.550	26.35	1.414	0.653	82.76	1.024	0.639	84.16	1.029
	Thrissur	0.932	46.31	1.036	1.493	42.10	0.961	1.5292	39.53	0.930

CONCLUSIONS

The following conclusions can be drawn from the study:

- The performance of the generalized wavelet based neural network (GWNN) models corresponding to the HG method for the estimation of ET_0 has been evaluated in this study. The GWNN models were developed by considering pooled daily climate data for a period of five years for four AERs in India.
- Further, the GWNN models were applied to models development and model testing locations to test the generalizing capability. The performance of developed GWNN models was compared with the GANN, GLR, GWR, and conventional methods using various performance indicators. Performance results of the GWNN models were very satisfactory as these models gave lowest RMSE and highest NSE values for most of the locations as compared to the GANN, GWR, and GLR models.
- During testing generalizing capability with the 15 model development locations and 10 model testing locations, the GANN and GWNN models performed better for most of the locations. Further, all GWNN, GANN, GWR, and GLR models performed much better than their corresponding conventional methods.

Literature cited

- Adamala, S., Raghuwanshi, N.S., and Mishra, A. (2015). Generalized quadratic synaptic neural networks for ET_0 modeling. *Environ. Process.* 2(2), 309-329 <http://link.springer.com/article/10.1007%2Fs40710-015-0066-6>
- Adamala, S., Raghuwanshi, N.S., Mishra, A., and Tiwari, M.K. (2014b). Development of generalized higher-order synaptic neural-based ET_0 models for different agroecological regions in India. *J. Irrig. Drain. Eng.* 140(12) [http://ascelibrary.org/doi/abs/10.1061/\(ASCE\)IR.1943-4774.0000784](http://ascelibrary.org/doi/abs/10.1061/(ASCE)IR.1943-4774.0000784)
- Cobaner, M. (2013). Reference evapotranspiration based on Class A pan evaporation via wavelet regression technique. *Irrig. Sci.* 13,119-134 <http://link.springer.com/article/10.1007%2Fs00271-011-0297-x>
- Evrendilek, F. (2014). Assessing neural networks with wavelet denoising and regression models in predicting diel dynamics of eddy covariance-measured latent and sensible heat fluxes, and evapotranspiration. *Neural Comput. Applic.* 24, 327-337 <http://link.springer.com/article/10.1007%2Fs00521-012-1240-7>
- Falamarzi, Y., Palizdan, N., Huang, Y.F., and Lee, T.S. (2014). Estimating evapotranspiration from temperature and wind speed data using artificial and wavelet neural networks (WNNs). *Agric. Water. Manage.* 140, 26-36 <http://www.sciencedirect.com/science/article/pii/S0378377414000869>
- Kisi, O. (2011). Evapotranspiration modeling using a wavelet regression model. *Irrig. Sci.* 29, 241-252 <http://link.springer.com/article/10.1007%2Fs00271-010-0232-6#page-1>
- Partal, T. (2009). Modelling evapotranspiration using discrete wavelet transform and neural networks. *Hydrol. Process.* 23, 3545-3555 <http://onlinelibrary.wiley.com/doi/10.1002/hyp.7448/abstract>